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THE LOBACHEVSKI PRIZE.

THE third award of the great Lobachevski prize was the occasion for considering particularly the achievements of two men during the past five years. The first of these is Professor Hilbert, of Goettingen; the second, Professor Barbarin, of Bordeaux.

The committee asked from the most distinguished of French mathematicians, Poincaré, a report on those works of Hilbert relevant to the decision.

With French thrift, Poincaré used as the greater part of this report a review of Hilbert's 'Grundlagen' which had already been published three times, two parts of which I quoted in my St. Louis address, to point out two errors (SCIENCE, N. S., Vol. XIX., No. 480, pp. 401–413). But the works to be considered included others which had only appeared after that review was written, so that Poincaré was compelled to recast and supplement this review of his, and some of these additions are of high interest.

Our ideas, he says, on the origin and scope of geometric verities have, since a century, evolved in a very rapid way. The creations of Lobachevski, of Bolyai, of Riemann have inaugurated a new era. Certes they have not discouraged the men, only too numerous, who seek to demonstrate the postulatum of Euclid. These, alas! nothing could discourage. But they have convinced all the true savants of the inanity of such an attempt. This was the first result of the invention of the non-Euclidean geometries. Lie pushed the

matter farther, but his spaces were all assumed by him to be number-manifolds.

A new progress remained to be accomplished, and the honor of this belongs to Hilbert. It is important, however, to say a word of the works which have prepared and rendered possible this advance.

Since the time of Lobachevski, mathematical thought has undergone a profound evolution, not alone in geometry, but in arithmetic and analysis. The notion of number has been made more clear and precise; at the same time it has received diverse generalizations. The most precious for the analyst is that which results from the introduction of neomonics, which modern mathematicians could not now dispense with.

Again, many Italian geometers, such as Peano and Padoa, have created a *pasigraphie,* that is to say, a sort of universal algebra where all reasonings are replaced by symbols or formulas.

Finally must be cited the book of Veronese on the foundations of geometry, where the author applies for the first time to geometry the transfinite numbers of Georg Cantor.

In 1899 Hilbert published a memoir entitled 'Grundlagen der Geometrie,' full of ideas the most original. Moreover, this was not the first time he had occupied himself with analogous questions, witness his letter of 1894 to Felix Klein: 'Ueber die gerade Linie als kürzeste Verbindung zweier Punkte.' He has since published in divers journals a series of articles entitled: 'On the theorem of the equality of the basal angles in the isosceles triangle'; 'New founding of the Bolyai-Lobachevski geometry'; 'On the foundations of geometry'; 'On surfaces of constant Gaussian curvature.'

All these articles have been united in a second edition of his jubilee memoir; and I must add that this second edition contains a series of improvements and additions which greatly augment its value.

It is, therefore, this second edition that we will follow in our analysis; but we will join with it, on the one hand, other works of Hilbert, such as his article 'Ueber den Zahlbegriff' and his Paris address on the mathematical problems of the future, and, on the other hand, many theses written by his scholars, under his direct inspiration, and which consequently aid us in comprehending his thought. The principal are: 'Ueber die Geometrieen in denen die Geraden die kürzesten sind' by Georg Hamel, and 'Die Legendre'schen Sätze ueber die Winkelsumme im Dreieck' by M. Dehn.

"Hilbert commences," continues Poincaré, "by establishing the complete list of axioms, straining not to forget one.

"Is his list final? It is permitted to believe so, since it seems to have been drawn up with care."

So says Poincaré for the fourth time. But if Hilbert's receiving the Lobachevski prize depended on his list of axioms being '*définitive,*' it could not be given to him. A young pupil of my own, R. L. Moore, by a charmingly simple proof has abolished the ugliest of the list, and Hilbert has already acknowledged the redundancy. Another point Hilbert himself changed in the French translation of his 'Festschrift' by Laugel. Poincaré had said: "The axiom that the sect AB is congruent to the inverse sect BA (which implies the symmetry of space) is not identical with those which are explicitly stated. I do not know whether it could be logically deduced from them; I believe it could."

In his 'Report,' Poincaré now says: "An important point is not here treated; the list of axioms should be completed by saying that the sect AB is congruent to the inverse sect BA.

"This axiom implies the symmetry of

space and the equality of the angles at the base in an isosceles triangle. Hilbert does not here treat this question, but he has made it the object of a memoir to which we will recur.'' This is a mistake. Hilbert explicitly assumes $AB \equiv BA$, but the equality of the basal angles does not follow therefrom.

It used to be supposed that the Euclidean straight was of essence continuous, and this putative continuity was rested upon to give continuity to the real number system. This mistake is made by Professor H. B. Fine in his book 'The Number-system of Algebra.' For example, on page 43 he says: *''The entire system of real numbers,* however, inasmuch at it contains an individual number to correspond to every individual point in the continuous series of points forming a right line, *is continuous.''*

Dedekind had long ago called attention to the fact that Euclid's space had no need of continuity. In an article of my own 'How the new mathematics interprets the old,' March 4, 1893, is a quotation of his construction of a discrete space, which goes on, ''yet despite the discontinuity, the perforation, of this space, all constructions occurring in Euclid are in it just as achievable as in perfectly continuous space. The discontinuity of this space would, therefore, never be noticed, never be discovered, in Euclid's science. Um so schoener erscheint es mir, das der Mensch ohne jede Vorstellung von messbaren Groessen, und zwar durch ein endliches System einfacher Denkschritte sich zur Schoepfung des reinen, stetigen Zahlenreiches aufschwingen kann; und erst mit diesem Hülfsmittel wird es ihm nach meiner Ansicht moeglich, die Vorstellung vom stetigen Raume zu einer deutlich auszubilden.''

There are naturally no points on the Euclidean straight to correspond to the series of irrational numbers, and Euclid felt no more ambition to have them there than he did to have a set of automobiles, and for the same reason, irrational numbers and automobiles had not yet been created.

Hilbert's axioms, analyzing Euclid's space, did not make it continuous. Poincaré called attention to this, and spoke of the space burdened with these irrational points as *our* space in contrast to Euclid's space, as if we were debarred by modernity from living in the splendidly free and disjointed space of the glorious old Alexandrian, who spurned the idea of any other way even for kings. ''In Hilbert's space,'' he says, ''there are not all the points which are in ours, but only those that one could construct, starting from two given points, by means of the ruler and compasses. In this space, for example, there would not exist, in general, an angle which would be the third part of a given angle.

''I have no doubt that this conception would have been regarded by Euclid as more rational than ours.''

He then proceeds, following Dedekind, to give an assumption which will lug in these irrational points. Hilbert in Laugel's translation did the same, but by a quite different assumption, which he calls the 'Axiom der Vollständigkeit.'

''Note. We remark that to the five preceding groups of axioms one may still add the following axiom which is not of a purely geometric nature and which, from the theoretical point of view, merits particular attention.

''AXIOM OF COMPLETENESS.

''To the system of points, straights and planes, it is impossible to add other beings (êtres) so that the system thus generalized forms a new geometry where the axioms of the five groups I.–V. are all verified. In other words: the elements of geometry form a system of beings which, if one maintains

all the axioms, is not susceptible of any extension.''

In speaking of the non-Archimedean geometry, the 'Review' made no mention of Veronese. The 'Report,' however, says: ''In this conception, so audacious, Hilbert had had a precursor. In his foundations of geometry Veronese had had an analogous idea. Chapter VI. of his introduction is the development of a veritable arithmetic and of a veritable geometry non-Archimedean where the transfinite numbers of Cantor play a preponderant rôle. Nevertheless, by the elegance and the simplicity of his exposition, by the depth of his philosophic views, by the advantage he has obtained from the fundamental idea, Hilbert has made the new geometry his own.''

The 'Report' is in 39 pages. The incorporation of the already published 'Review' stops with page 25. The last fourteen pages are entirely new, as follows: The memoir we have just analyzed puts in evidence the importance of the new non-Archimedean geometry. It discusses the rôle of the axiom of Archimedes in geometric reasoning; and the principal result of this discussion may be summed up thus: If we abandon this axiom and retain only the axioms of the first four groups, the essential results of Euclidean geometry are not altered; but this is not so if one retains only the projective axioms [assumptions of association] and those of order [betweenness], together with the postulatum of Euclid, but abandons at the same time the axiom of Archimedes and the metric axioms [assumptions of congruence]; we come then to the non-Pascalian geometry.

Then comes the question, does this that we have just said of the Euclidean geometry remain true of the Lobachevskian?

In other words, if we preserve only the axioms of the first three groups (projective, of order and metric) and replace the postulatum of Euclid by that of Lobachevski, shall we arrive at the fundamental theorems of Lobachevski *without using the axiom of Archimedes?*

This is the question that Hilbert has settled in his article 'Ueber eine neue Begründung der Bolyai-Lobatschefskyschen Geometrie.'

He answers it affirmatively and shows in particular that there always exists a common perpendicular to two straights of the plane which do not meet without being parallel.

I would call attention to the statement of the postulate of Lobachevski: ''If b is any straight of the plane and A a point not situated on this straight, there pass always through A two demi-straights [rays], a_1 and a_2, which are not in the prolongation one of the other and which do not cut the straight b, while every semi-straight passing through A and situated in the angle formed by a_1 and a_2 meets b.''

''It is these two demi-straights, a_1 and a_2, which have received the name of *parallels.*'' They do not meet the straight b, but they serve as *limit* to the angle wherein are found the straights which meet b and to the angle wherein are found the straights which do not meet b.

I would signalize an elegant theory of what might be called the points at infinity of the Lobachevskian plane, and of which the laws are the same as those of the addition and the multiplication of real numbers. One may draw thence a very simple and very suggestive exposition of the non-Euclidean geometry.

The origin of our acquaintance with the theory of parallels is found in the theorems of Legendre which establish a necessary correlation between the sum of the angles of a triangle and the choice between the three geometries, Euclidean, Lobachevskian, Riemannean.

What rôle does the axiom of Archimedes

play in these theorems? This question interested Hilbert and under his inspiration M. Dehn has made it the subject of a thesis which I can not pass over in silence. The conclusions of Dehn show that without the axiom of Archimedes the theorems of Legendre are no longer true.

It is still true that if *one* triangle has the sum of its angles equal to (or greater than) (or less than) two right angles the same is true of all the others. It is still true that if this sum is less than two right angles, one can draw many parallels to a straight through one point. It is true that if it is greater than two right angles, the postulatum of Euclid is false, and that if it is equal to two right angles it is impossible that two straights always meet, but *the other theorems of Legendre are not true.*

There exists a plane geometry where, the sum of the angles of a triangle being greater than two right angles, one can draw to a straight through one point an infinity of parallels (so I call straights which do not meet); this is the *non-Legendrean* geometry.

A geometry exists where the sum of the angles is equal to two right angles, and where one can draw to a straight through one point an infinity of parallels. This is the *semi-Euclidean geometry.*

It will suffice to explain here what this latter is, the former being altogether analogous. For this it is necessary to return to what I have said of the non-Archimedean geometry. I have explained how the non-Archimedean plane is deduced from the ordinary plane by the adjunction of new points; how for deducing a non-Archimedean straight D_1 from the ordinary straight D_0, it is necessary to add to it:

1. On the one hand, an infinity of new points between every two demi-straights S' and S'' of which the totality forms D_0.

2. On the other hand, an infinity of new points to the right of all the ordinary points of D_0, and an infinity of new points to the left of all the ordinary points of D_0. Well, retain the new points of the first sort, that is to say, those which are at a finite distance, and suppress the new points of the second sort, that is to say, those which are at an infinite distance.

Then let D be any straight and A any point; then there will be an infinity of straights passing through A and which do not meet D, those, namely, which would have met it in one of the new points of the second sort, if these points had not been suppressed. However, all the theorems of Euclid remain and every rotation or every translation will transform into itself the non-Archimedean plane so mutilated.

It seems that here is a contradiction with the results of the article just cited: 'Ueber eine neue Begründung. * * *'

If, as Hilbert has shown, the geometry of Lobachevski can be deduced from his postulate without the intervention of the axiom of Archimedes, how can there be a geometry semi-Euclidean, that is to say a geometry where the theorems of Euclid accord with the postulate of Lobachevski?

It seems that this difficulty springs from this, that the enunciation of the postulate is not the same in the two cases.

Dehn assumes that through a point one can draw an infinity of straights which do not meet a given straight, and an infinity of straights which meet it.

The first form an ensemble E_1, the others form an ensemble E_2. Hilbert supposes, in addition, that there exists a limiting straight which appertains to the ensemble E_1, and such that every straight comprised between this limit straight and a straight of E_2 appertains likewise to E_2. It is this limit straight which Hilbert considers as the *parallel properly so called.*

In the geometry of Dehn this parallel

properly so called does not exist. Here would be an interesting question to examine closely. Is it possible to construct a non-Archimedean geometry where this parallel properly so called exists and to which are applicable the results of Hilbert?

An analogous question is treated in another article by Hilbert, 'Ueber die Gleichheit der Basiswinkel im gleichschenkligen Dreieck.'

In the ordinary plane geometry, the plane is symmetric, which expresses itself by the equality of the angles at the base of the isosceles triangle.

One should make this *symmetry of the plane* figure in the list of metric axioms. In all the geometries more or less strange of which we have hitherto spoken, in those at least where one admits the metric axioms, in the metric non-Archimedean geometry, in the new geometries of Dehn, in those which have made the subject of the memoir 'Ueber eine neue Begründung * * *,' this symmetry of the plane is always assumed. Is it a consequence of the other metric axioms? Yes, as Hilbert shows, if one admits the axiom of Archimedes. No, in the contrary case.

There are non-Archimedean geometries where all the metric axioms are true, with the exception of this of the symmetry of the plane. Here is an example:

The non-Archimedean numbers previously defined may be infinite or finite or infinitesimal; but an angle will be always finite or infinitesimal because of the relation

$$\cos^2\varphi + \sin^2\varphi = 1.$$

An angle may, therefore, always be put under the form $\theta + \tau$, θ being an ordinary real number and τ a non-Archimedean infinitesimal.

We define then the rectangular coordinates of a point, the straights and the translations in the ordinary manner and define rotation in the following manner. Let α, β be the coordinates of the center of rotation; $\theta + \tau$ the angle of rotation; x, y the coordinates of any point before the rotation; x', y' its coordinates after the rotation; one will have

$$(x' - \alpha) + i(y' - \beta) = e^{(i\theta+\tau+i\tau)}[(x - \alpha) + i(y - \beta)].$$

Consider the group formed by the rotations about the origin. This group will not be permutable for the transformation which changes y into $-y$, nor for any transformation which retains the origin, which changes straights into straights and of which the square reduces to the identical transformation. *The plane is, therefore, not symmetric.*

All the other metric axioms subsist, however, as does also the postulatum of Euclid and even a new axiom which Hilbert calls *Axiom der Nachbarschaft* and which he states thus:

"Given any sect AB, one can always find a triangle in the interior of which can be found no sect congruent to AB."

This results easily from the equation of the circle. The equation of a circle of radius ρ and center α, β is in fact:

$$(x-\alpha)^2 + (y-\beta)^2 = \rho^2 e^{2\tau};$$

$$\frac{y-\beta}{x-\alpha} = \tan(\theta + \tau).$$

In return, it is not true that the angles at the base of an isosceles triangle are equal; it is not true that in a triangle one side is less than the sum of the other two; finally the theorem of Pythagoras on the square of the hypothenuse is not true.

"It is for this reason that this geometry is called *non-Pythagorean*. [I may interpolate here that Poincaré is in error in saying Hilbert shows that the equality of the basal angles can be proved from the other metric axioms if one admits the axiom

of Archimedes. In addition the new Axiom der Nachbarschaft is used.]

"I come to speak now of an article entitled 'Ueber die gerade Linie als kürzeste Verbindung zweier Punkte' that I can not separate from a thesis on the same subject, written by Hamel under the inspiration of Hilbert. Here we are less far from home; not only is there no question of renouncing the axiom of Archimedes, but we encounter only analytic functions which may be differentiated and integrated.

"Suppose that one has defined straights in the ordinary fashion and that one admits the projective axioms, those of order and the theorems of Desargues and Pascal. Define now the length of an arc of any curve; it is not necessary to choose this definition so as to satisfy the metric axioms, that is to say so as to render possible the movement of a rigid figure.

"Is it possible so to make this choice that the straight line shall be the shortest path from one point to an other?

"The definition of the straight is not changed, but that of the circle is in a very large degree arbitrary; it is only necessary that all the circles which have their center on a straight and which pass through a point of this straight have at this point the same tangent. The problem permits an infinity of solutions.

"Minkowski, for an arithmetic purpose, has developed one of them where all the circles are curves similar to each other in the ordinary sense of the word. Hilbert, from 1894, had given another of them which may be thus characterized: We consider a connected closed curve which will serve as fundamental curve. Let D be a straight, M a point of this straight; all the circles which have their center on D and which pass through M have the same tangent T, and this tangent, when the point M describes the straight D, pivots around a fixed point which is the intersection of two tangents to C at the points where this curve is cut by the straight D.

"Finally Hamel has in his thesis given the general solution of the question, but this solution is too complicated to be expounded in few words.

"I arrive at an important memoir of Hilbert's which is entitled 'Grundlagen der Geometrie,' which bears, consequently, the same title as his 'Festschrift,' but where, however, he places himself at a point of view altogether different.

"In his 'Festschrift,' in fact, as one sees by the preceding analysis, the relations between the notion of space and the notion of group, as they result from the works of Lie, are left to one side or relegated to a secondary part. The general properties of groups do not appear in the list of his fundamental axioms.

"This is not so in the memoir of which we are about to speak. Hilbert assumes a plane about which he makes the following hypotheses:

"1. The points of this plane correspond one to one to the points of the ordinary plane or to a part of these points. Thus each point of the new plane has its correspondent in the ordinary plane; but there may be on the ordinary plane points which have no correspondent on the new plane.

"The new plane has, therefore, so to say, less points than the ordinary plane, which is the contrary of that which happened for the non-Archimedean plane. The points of the ordinary plane which have correspondents on the new plane are called Bildpunkte. The ensemble of Bildpunkte forms on the ordinary plane a region which Hilbert assumes continuous and connected in such fashion that around each point of this region one can describe a circle of radius sufficiently small to be contained in this region and that one can go from one point to the other of the region, in follow-

ing a continuous curve and without going out from the region.

"2. The points of this new plane are susceptible of transformations called *movement* and which form a *group*.

"3. Among these movements, there are an infinity which leave fixed a certain point M and which are called rotations about M.

"The ensemble of the transformés of the same point A by all these rotations is called a circle. Every circle has an infinity of points.

"4. The group of movements forms a *closed system;* which means this: if there are an infinity of movements which change two points A_0 and B_0 the first into A_1 and B_1, the second into A_2 and B_2, $\cdots$ the nth into A_n and B_n; and if the point A_n tends towards A and the point B_n towards B when n increases indefinitely, there will also be in the group a movement which will exactly change A_0 into A and B_0 into B; and the same holds if in place of two points we consider three of them or only one.

"I have slightly abridged the statements, making them lose, it is true, a little of their precision, but without taking away anything essential. About these enunciations I have certain observations to make.

"The question in brief is to find all the groups of transformations of the plane into itself, or of a part of the plane into itself, these groups being subjected only to conditions in appearance very slightly restrictive. How, therefore, can one arrive at conclusions so precise?

"This results from the definition which Hilbert gives of movement. To be a movement, a transformation must satisfy many conditions; first it must be continuous and transform two points infinitely near into two points infinitely near; then it must be biuniform, that is to say, that every point of the plane must have one transformé and only one, and be the transformé of one point and of only one.

"By that are found to be excluded a very great number of groups; for example, the group of the projective transformations of the plane and the group of homothetics, that is to say, transformations which change every plane figure into a homothetic figure [a figure similar and similarly placed].

"Why? If we take, for example, the homothetic group we see that it contains degenerescent transformations, those where, the center of homothety moreover being any whatever, the ratio of homothety is nul or infinite. In these transformations the center of homothety has an infinity of transformés or is the transformé of an infinity of points. These degenerescent transformations, without which the group would not be a *closed system,* can not be excluded, nor any more can they be retained, since they do not satisfy the definition of movement.

"One may see in the same manner that a circle can not contain all the points of the plane, otherwise, among the rotations about the center of this circle, there would be one which would bring to the center a point of the plane, other than the center, so that the center would be the transformé of two points, of this point and of itself.

"That implies the existence of an invariant analogous to distance.

"One sees that the conditions are more restrictive than they seemed. In relation to the ideas of Lie, the progress realized is considerable. Lie supposed that his groups were defined by analytic equations.

"The hypotheses of Hilbert are much more general.

"Without doubt this is not yet entirely satisfactory, since if the *form* of the group is supposed any whatsoever, its *matter,* that is to say, the plane which undergoes the transformations, is still obliged to be a

Zahlenmannigfaltigkeit in the sense of Lie. This is, nevertheless, a step in advance, and besides Hilbert analyzes better than any one had done it before him the idea of Zahlenmannigfaltigkeit, and gives hints which may become the germ of an axiomatic theory of analysis situs.

"I can here only summarize the general march of Hilbert's ideas.

"He shows first that the points of a circle can be arranged in a determined circular order and that this order is not altered by rotations; then he shows that this order falls into the same *type of order* as the corresponding order of the ordinary circle, that is to say, into the type of the continuous. Thence he deduces this consequence that the circle is a continuous closed curve, because it must correspond point for point to the ordinary circle.

"One sees then that if a rotation does not displace one point of a circle, it will not displace any other point of this circle. Thence one can deduce that if a rotation does not displace one point different from its center, it will not displace any of the points of the plane and will reduce to identity. From this results finally that the group of rotations around a point M has the same structure as the group of ordinary rotations.

"One sees at the same time that there is no movement which leaves fixed two points of the plane, and that we can pass by rotations from one point of the plane to any other point whatsoever of the plane.

"All these demonstrations are extremely delicate; they require the repeated employment of the theorems of Cantor.

"This is to say that they are perforce very long and that the goal which one perceives immediately and which one thinks to touch can be attained only by long efforts.

"The greatest step is then accomplished; now we know that our group derives from certain subgroups, those of rotations, of which we know the structure, and this structure makes these subgroups fall into the category of Lie's continuous groups.

"We have, therefore, only a few difficulties still to vanquish, but Hilbert wishes first to define the straight and he has done it in a very original fashion.

"He rejects first the projective definitions of the straight which require considerations foreign to his premises. On the other hand, his geometry is a *plane geometry*.

"If we may use space of three dimensions, the theory of groups leads us naturally to a very simple definition of the straight, considered as axis of rotation; but here we can not use this, since we can not go out of the plane.

"Hilbert follows wholly another way. Let there be two points, A and B; define the middle of these points, that is to say the center of a rotation which changes A into B and B into A. Hilbert begins by demonstrating that two points have always a middle and have only one. It is here that comes in an hypothesis which above must have astonished the reader; we have supposed that the last axiom (which one states in an abridged fashion in saying that the group of movements is a closed system) is applicable not only if one envisages two points A_0 and B_0, but also if we consider three points. We have, therefore, made an hypothesis more restrictive than if we had limited ourselves to the consideration of two points A_0 and B_0. Was this restriction really necessary?

"It is in this part of the theory that it plays its rôle. We consider an infinity of points $B_1, B_2, \cdots B_n$, and the middles $M_1, M_2, \cdots M_n, \cdots$ of the sects $AB_1, AB_2, \cdots AB_n$; when n increases indefinitely, B_n tends toward B and M_n toward M and we make use of the hypothesis in question to show that M is the middle of AB. Had

it been impossible to use it, we could have been sure of this only after having constructed a special pseudogeometry.

"As it is, two points A and B being given, Hilbert constructs the middle of the sect AB, then the middle of the two sects MA and MB and so on. He thus obtains an infinity of points which form an ensemble E; he considers the derivative of this ensemble E, that is to say the assemblage of the limiting-points of E, points such that in any interval containing one there are an infinity of points of E. He shows that this derivative is a continuous line, and it is this line that he calls the straight [die wahre Geradê].

"The fundamental principles of the ordinary Euclidean or non-Euclidean geometry may then be easily established and in particular the metric axioms.

"It is impossible not to be struck by the contrast between the point of view where Hilbert places himself here and that which he had adopted in his 'Festschrift.' In this 'Festschrift' the axioms of continuity occupy the last rank and the grand affair was to know what geometry became when one threw them aside. Here on the contrary it is continuity which is the point of departure and Hilbert is especially preoccupied to see what one gets from continuity alone, joined to the notion of the group.

"It remains for us to speak of a memoir entitled 'Flächen von konstanter Krümmung.'

"It is known that Beltrami has shown that there are in ordinary space surfaces which are the image of the Lobachevskian plane, namely the surfaces of constant negative curvature; we know what an impulse this discovery gave to the non-Euclidean geometry. But is it possible to represent the whole entire Lobachevskian plane on a surface of Beltrami without singular point?

"Hilbert demonstrates that it is not; he founds his proof on the following theorems relative to the Beltrami surfaces.

"A quadrilateral formed of asymptotic lines has its opposite sides equal.

"The surface of a polygon formed of asymptotic lines is proportional to the spherical excess and it is the same with the surface of a polygon formed of geodesic lines; only in the first case the spherical excess is positive, in the second case it is negative.

"The author shows then that on a Beltrami surface without singular point one can not trace a closed asymptotic line; that an asymptotic line can neither cut itself, nor cut another asymptotic line in more than one point. Every other hypothesis would lead to polygons of which the spherical excess would be nul. Thence it follows as a consequence that if such a surface corresponds point for point to the non-Euclidean plane, this correspondence must be biuniform. But then in evaluating the total surface starting from the area of the polygon formed of asymptotic lines or from the area of the geodesic polygon we find in the first case that this total surface is finite, in the second that it is infinite. This contradiction demonstrates the theorem enunciated.

"In that which concerns the surfaces of positive constant curvature, to which the geometry of Riemann corresponds, Hilbert demonstrates that aside from the sphere there is no other closed surface of this sort. In fact, if we consider a portion of surface of constant positive curvature, the maximum of the great radius of curvature can not be attained *in the interior* of this portion, but only on its contour. This is a proposition entirely analogous to a well-known theorem relative to the potential.

"It follows thence immediately that if the surface is closed, the maximum can be nowhere attained and the radius of curva-

ture is constant. Thus we easily come back again to the sphere.

"After this analysis, all commentary is useless. One sees at how many different points of view Hilbert has placed himself, how profound is his analysis.

"His works mark an epoch and he seems entirely worthy of the Lobachevski prize. —Poincaré."

The 'Report on the works of Monsieur Barbarin, professor of higher mathematics at the Lyceum of Bordeaux, relative to the non-Euclidean geometry,' is by Professor Mansion, of Ghent, as follows:

"I. *List of the Works of M. Barbarin.* —M. Barbarin has published, from 1898 to 1902, the following memoirs and works relative to the non-Euclidean geometry.

"1. Géométrie générale des espaces (*Association française pour l'avancement des sciences.* Congrès de Nantes, 1898, pp. 111–132).

"2. Propriétés angulaires des cercles focaux dans les coniques (*Ibid.*, 1898, pp. 132–139).

"3. Constructions spheriques à la règle et au compas (*Mathesis,* 1899, pp. 57–60; 81–85).

"4. Etudes de géométrie analytique non-euclidienne (*Mémoires courounnés et autres Mémoires* publiés par l'Académie royale de Belgique, 1900, t. LX., 167 pp. in 8°. This memoir was presented to the Royal Academy of Belgium, the fourth of December, 1897).

"5. Le cinquième livre de la Métagéométrie (*Mathesis,* 1901, pp. 177–191).

"6. Les cosegments et les volumes en géométrie non euclidienne (Extrait des mémoires de Bordeaux, 1901; 20 pp. in 8°).

"7. La géométrie non euclidienne. Paris, Naud, February, 1902 (collection *scientia,* 79 pp. in 12°).

"8. Bilatères et trilatères en Métagéométrie (Mathesis, 1902, pp. 187–193).

"9. Polygones réguliers sphériques et non euclidiens (*Le Matematiche pure ed applicate,* 1902, t. II., pp. 137–145).

"We will now analyse these works, classing the results found by M. Barbarin under three heads:

"*Elementary geometry, conics and quadrics, infinitesimal geometry.*

"II. *Elementary geometry.*—In his little book entitled 'The non-Euclidean geometry' (List No. 7), M. Barbarin expounds the first principles of the geometry, especially after Saccheri, Bolyai and Lobachevski and, among the moderns, DeTilly, Gérard, Mansion. But, besides, he makes known, whether in this little book or in divers special notes, results which are his own.

"1. *Bilaterals and trilaterals* (List No. 8).—The author proves in an elementary manner, *without recourse to analysis,* that the locus of points equidistant from two straights is a straight; that the bisectors, the medians and the altitudes of a trilateral are copunctal (meet in the same point) [real at a finite or infinite distance, or ideal], even if the vertices of the trilateral are all or in part reals at infinity or ideals.

"He deduces from the theorem on the three altitudes a novel construction of the normal common to two Lobachevskian straights which only meet at an ideal point.

"2. *Fundamental constructions* (List No. 3, No. 4, § I., pp. 5–14; No. 7, pp. 46–49).—M. Barbarin gives the means of constructing, with the ruler and the compasses, a right-angled triangle or a trirectangular quadrilateral given two elements and thence deduces all the fundamental constructions of the non-Euclidean geometry.

"He depends, in these constructions, on the old theorems of Lobachevski and Bolyai and on three new theorems which seem to have escaped these illustrious geometers. Here they are, in Lobachevskian geometry:

"Let *ABCD* be a quadrilateral trirect-

angular in B, C, D, and having, consequently, angle DAB acute; E — a point situated between D and A, F — a point situated between B and A, such that $CE = BA$, $CF = DA$.

"We have (1) $BF = DE$. (2) CE is asymptote (Lobachevskian parallel) to BA, CF to DA. (3) The perpendiculars let fall on CB, CD from points equidistant from C taken on CE, CF intersect on the diagonal CA.

"3. *Regular spheric and non-Euclidean polygons* (List No. 9.—M. Barbarin has found constructions simple and novel for the regular polygons of 3, 6, 5, 10, 15 sides, applicable at the same time in Euclidean geometry and in non-Euclidean geometry of Riemann and of Lobachevski, *for the sphere and for the plane.*

"4. *The fifth book of Metageometry* (List No. 4, ch. IV., pp. 94–99, No. 5, No. 7, pp. 37–41).—M. Barbarin calls fifth book of 'Metageometry' that which corresponds to the fifth book of the 'Elements of Legendre' or to the eleventh of Euclid.

"He makes an elementary exposition of it more complete than does any of his predecessors; here and there it could have been intuitive if he had depended more on the asymptotic property of Lobachevskian parallels. There is room to cite in this work the two following theorems: (1) That a right angle may be projected upon a plane into a right angle, it is necessary and sufficient that the projector of the vertex be normal to the plane and to one of the sides of the angle.

"(2) Two Riemannean straights not situated in the same plane have two common normals; if these normals are equal, the two straights are equidistant.

"Descriptive non-Euclidean geometry rests on the first proposition.

"From the second, it results that there exist, in Riemannean geometry, skew squares and rectangles having four right angles and surfaces equidistant from a straight with rectilinear generators.

"5. *Coordinates; geometry of n dimensions* (List No. 1, No. 4, § II., pp. 14–28, § IV., pp. 84–101).—In his 'Studies in non-Euclidean Analytic Geometry,' M. Barbarin has been led to certain new developments of the theory of coordinates, and, consequently, to expound by the calculus, the fundamental properties of the straight and of the plane, of angles and of distances, of the circle and of the sphere.

"This is, in the main, under analytic form, the complement of his other studies on the 'Elements.'

"The memoir of pure analysis, entitled 'General Geometry of Spaces' is a generalization of the formulas of Euclidean or non-Euclidean geometry of three dimensions relatively to the straight, to the plane, to the triangle and to the trihedrals and the trigonometric relations relative to them, when one considers a variety of n dimensions. The author shows, in particular, that for such varieties there exists also a limit-case that we may call *Euclidean geometry of n dimensions.*

"III. *Conics and quadrics* (List No. 2, No. 4; the essential part of 2 is reproduced in 4). The 'Studies in non-Euclidean Analytic Geometry' constitute M. Barbarin's largest work. It is devoted, for the major part (§ III., pp. 29–84; § V., pp. 101–139), to a classification of conics and quadrics more complete than that of his predecessors, without having any recourse to the Cayleyan geometry.

"A. *Conics.*—The author first reduces to its most simple forms the general equation of the second degree or rather a ternary quadratic form.

"In Riemannean geometry he finds only two kinds of curves: *imaginary conic* and *ellipse,* the latter having the *circle* as variety.

"The Lobachevskian geometry is much

richer in curves of the second degree. The curves with real center are the *ellipse* (real, semi-real, ideal or imaginary) with the important varieties, *circle* and *hypercycle* (= equidistant from the straight). The curves denuded of center, even at infinity are the parabolas (elliptic, veritable, hyperbolic). The curves with center situated at infinity are the *oriconics* (oriellipse, with the variety oricycle of Lobachevski, orihyperbola).

"M. Barbarin generates the lines of the second degree by homography and by movements of linkages; he investigates their foci, their focal circles and their directrices; finally their reciprocal curves. Two reciprocal curves of the second degree are such that each is the locus of the center (real or ideal) of the tangents of the other. The properties of these curves are a consequence of the principle of duality, which is, so to say, evident in non-Euclidean geometry.

"The author then studies the plane sections of a cylinder or of a cone of the second degree (that is to say, having for plane directrix a curve of the second degree); he finds again in this way all the varieties of curves of the second degree, which are, therefore, truly conics. He extends to non-Euclidean conics the most celebrated theorems relative to Euclidean conics and, in particular, those of Dandelin. He obtains in a manner more systematic still all the curves of the second degree, Riemannean, Euclidean, Lobachevskian, by cutting the cone of the second degree by a concentric sphere, the common center being real, at finite or infinite distance, or ideal.

"One again finds conics in cutting by a plane the straight equidistant surface (tube of revolution with rectilinear axis, or hypercycloide of revolution). In Riemannean geometry there is a case where one finds as section two straights equidistant from the axis, but not coplanar with the axis (compare above, II., 5): these straights are the *helices* of this surface.

"B. *Quadrics.*—The reduction of the general equation of the surfaces of the second degree is deduced from the discussion of the equation in s of the nth degree.

"In Riemannean space, we find two principal species—*ellipsoid* (with the varieties ellipsoid of revolution, tube sphere), *pipe-hyperboloid* (with the varieties cone, hyperboloid of revolution or elliptic tube, two planes).

"In Lobachevskian space we find first the species *ellipsoid* (with three unequal real axes, semi-real with two real axes, with one real axis or imaginary); the *first hyperboloid* (with one nappe real, with two nappes real, with one nappe ideal); the *second hyperboloid* (with two nappes real or ideal). The varieties or limits of these three species are very numerous. All these surfaces have a center and three principal planes.

"The *paraboloids* (elliptic, semi-elliptic or hyperbolic) and their numerous varieties have no center and have two principal planes. They *cut* the sphere of infinite radius.

"At the limit, when they become *tangent* to this sphere, they are transformed into oriquadrics (oriellipsoid, orihyperboloid) and into their varieties.

"M. Barbarin has studied the rectilinear and circular sections of these surfaces, their focal spheres and their directrices.

"IV. *Infinitesimal geometry.* 1. *Measure of areas and of volumes* (List No. 4, pp. 164–167; No. 6; No. 7, pp. 50–59).—M. Barbarin in his little book, 'The non-Euclidean Geometry,' summarizes in some pages the results found by Lambert, Lobachevski, Simon, etc.; but he calls attention also (List No. 6) to an original idea of which he is the author. He has remarked that the volume of a frustum of a non-Euclidean cone of revolution is propor-

tional to the difference between the projection of the generatrix on the axis and this generatrix multiplied by the cosine of the angle which it makes with the axis if it meets it, or of the normal common to the generatrix and axis. He has thence deduced a *general formula for volumes decomposed into infinitesimal spindles of revolution.*

"By means of this formula he has been able to reach a number of known results; especially he has been able to make an advance in the very difficult question of the volume of the tetrahedron, which, as is known, has arrested Gauss, Bolyai and Lobachevski and all their successors.

"M. Barbarin finds an expression for the volume of the tetrahedron *where are introduced naturally the products of the edges by the corresponding dihedrals.* To achieve the solution of the question, it is requisite to find, under a finite form, certain functions relative to the faces which present themselves in the calculations under the form of integrals or complicated series.

"2. *Geodesic lines of tubes and pseudospheres* (List, No. 4, pp. 139–164).—In the last section of his 'Studies in non-Euclidean Analytic Geometry' the author has reached one of the most beautiful theorems of metageometry.

"It has been known, since Lobachevski and Bolyai, that the characteristic geometry of orispheres is Euclidean; since Beltrami, that of the Euclidean pseudosphere is Lobachevskian; finally it is evident that that of the sphere is Riemannean.

"The theorem of Barbarin (it is to be hoped that it will retain this name) comprises and generalizes in the most unexpected manner these particular propositions. Here it is in its most condensed form: *Each of the three spaces, Euclidean, Lobachevskian, Riemannean, contains surfaces of constant curvature of which the geodesic lines have the metric properties of the straights of the three spaces.*

"These surfaces are the spheres (characteristic geometry, Riemannean); the tubes or surfaces equidistant from a straight, it being possible for the distance to be infinite, which gives the orispheres (characteristic geometry, Euclidean); finally the pseudospheres, that is to say the surfaces of revolution which have for meridians a tractrix or line of equal tangents (characteristic geometry, Lobachevskian).

"The property of the surfaces equidistant from a straight is almost evident and has been found also by Whitehead; but the existence of Lobachevskian tractrices and pseudospheres and above all of Riemannean and the properties of their geodesics were not suspected before M. Barbarin.

"The curvature of the pseudospheres is negative in Riemannean space as in Euclidean space; it is negative, nul or positive in Lobachevskian space.

"V. *Résumé and Conclusion.*—Non-Euclidean geometry owes to M. Barbarin (1) fundamental properties of the plane trirectangular quadrilateral; (2) the discovery of Riemannean equidistant straights; (3) the complete classification of non-Euclidean conics and quadrics; (4) the most intuitive formula that we know for the determination of volumes, with a remarkable application to the tetrahedron; (5) finally and above all the beautiful general theorem cited above on the geodesics of tubes and pseudospheres, in the three geometries.

"All these results have been obtained by the direct study of the figures without borrowing anything from the Cayleyan geometry.

"If Lobachevski should come back to the world, he would recognize in M. Barbarin

a worthy continuer of his work, inspired with his ideas and his method.

"We believe, therefore, that the Physico-mathematic Society of Kazan may legitimately decree to him the Lobachevski prize. —P. Mansion."

To Professor Hilbert I am particularly, personally indebted. My 'Rational Geometry' is an attempt to give every teacher, every scholar the benefit of coming after him, the priceless advantage of living since the outpouring of his genius.

Monsieur Barbarin has honored me with his genial friendship.

I suppress the temptation to institute comparisons or discuss a decision.

GEORGE BRUCE HALSTED,
Membre d'honneur du Comité Lobatchefsky.

THE TROPHOBLAST: A REJOINDER.

THE name *Trophoblast* was used for the first time by me in the meeting of the Anatomical Congress at Würzburg in 1888, and its earliest definition is found in the report of that meeting in Nos. 17 and 18 of the *Anatomischer Anzeiger,* Bd. III. We there read, concerning a very early stage of the hedgehog (p. 510):

"Die äussere Wand der Keimblase ist verdickt (drei bis vierschichtig) und besitzt wabige Lacunen. Für diese äussere (epiblastische) Schicht sei der Name *Trophoblast* gewählt."

In a footnote we find in addition (p. 511):

"Es ist meiner Ansicht nach zweckmässig, sich bei der Säugethier-embryologie diesen Namen zu wählen, um damit den nicht zum Aufbau des Embryos verwendet werdenden Epiblast anzudeuten * * *."

It is evident from the citations here given that the names outer epiblastic wall of the mammalian blastocyst and trophoblast are synonyms. Later researches have been directed towards the question how in other mammals than the hedgehog the separation between the epiblast of the embryonic shield, *i. e.*, the formative epiblast and the trophoblast, comes about.

In the same Bd. III. of the *Anatomischer Anzeiger,* on p. 907, mention is again made of the hedgehog's 'geschlossene Trophoblastblase (wie ich den primären Epiblast, von dem sich durch Abspaltung der Epiblast des Fruchthofes nach innen abhebt, zu benennen vorschlug).'

Again, in the article on the placentation of *Erinaceus* in Vol. 30, Pt. 3 (1889), of the *Quart. Journ. of Microsc. Science,* where the definition was reproduced, it is insisted upon (p. 298) that "the use of the name trophoblast will render unnecessary such circumlocutory expressions as 'outer epiblastic layer of the blastocyst,' 'primitive exochorion,' etc." Further argumentation on p. 299, in which the allantoidean and the omphaloidean trophoblast is defined, leaves not the faintest doubt as to what the name *trophoblast* has originally stood for.

Five years later (1894), in an article, 'Spolia nemoris,' which appeared in Vol. 36 of the *Quart. Journ. of Micr. Science,* I again insisted (p. 111) that 'new and valid reasons are thus accumulated for designating the outer layer of precociously segregated epiblast cells that form the wall of this vesicle [the early mammalian blastocyst] by a separate name, [for which] I have proposed the name of trophoblast.' Somewhat further is added (p. 112): 'in *Tupaja* and *Tarsius* portions of the trophoblast undergo very active proliferating processes preparatory to the placentary fixation of the blastocyst, whereas in my former papers I have described the same activity for *Erinaceus* and *Sorex.*'

Finally, in 1895 (*Verhandl. der Kon. Akad. v. Wetenschappen te Amsterdam,* Vol. IV., No. 5, p. 18), I reaffirm that:

'die von mir Trophoblast genannte Keimschicht ist * * * die äussere schicht der Säugethierkeimblase welche vor der definitiven Ausbildung des formativen Epiblastes dieses sowie die Hypoblastanlage umhüllt und an der Bildung des Embryos überhaupt keinen Antheil nimmt.'

In this last paper I have for the first time asserted that in my opinion the Sauropsidan arrangement, as well as that of the Ornithodelphia, can not possibly be looked upon as ancestral to what we find in the monodelphic (and didelphic) mammalia and that, on the contrary, the trophoblast (*l. c.*, p. 57, No. 7) is a precociously segregated larval envelope which encloses an inner cell mass, out of which the embryo is going to be built up. I have at the same time drawn a comparison between the mammalian trophoblast and the 'Deckschicht' in amphibian development and have also drawn attention to those cases where remnants of a trophoblastic layer could be detected in the Sauropsida.

Only in 1902, however, have I gone yet further back, and leaving the recent amphibia out of the ancestral line, I have attempted to draw a comparison between the trophoblast (and the other fœtal membranes coexistent with it) of the Amniota and larval envelopes of invertebrate predecessors (*Verhandl. der Kon. Akad. v. Wetenschappen te Amsterdam,* Vol. VIII., No. 6, 1902, p. 53). Alinea! It has now been shown that since the first introduction of the name *trophoblast* sixteen years ago my own definition and interpretation of it has not undergone any alteration, although advances have been made in the appreciation of its theoretical significance.

And it is for this reason difficult for me to understand that the name has been misunderstood both by embryologists and by gynæcologists, even to such an extent that the writing of the present article seems necessary to prevent further confusion.

So, for example, Charles S. Minot's definition of the trophoblast on p. 100 of his 'Laboratory Text-book of Embryology' (1903) as 'a special layer of cells developed on the outer surface of the ectoderm of the mammalian blastodermic vesicle' is both wrong and misleading. Several statements in the same paragraph on p. 107, *e. g.*, that the trophoblast is sometimes developed only later; that it disappears when the placenta is being formed, etc., are likewise in complete disaccordance with the original definition, such as it was substantiated by the different quotations given above.

In attempting to explain for myself how Minot can have fallen into this error—from which consultation of the papers above quoted would have withheld him—I can not but suppose that Bonnet's 'Grundriss der Entwickelung der Haussäugethiere' must have led him astray. In this we find on p. 31 a woodcut (Fig. 17), in which the trophoblast (Bonnet's primärer Ectoblast) is represented as a separate layer outside of the ectoderm of the monodermic blastocyst of the hedgehog and which woodcut is marked 'nach Hubrecht,' although I never published anything of the kind, nor in my writings have ever, as we have seen above, given the slightest justification to an interpretation so entirely inconsistent with my own views which have repeatedly been expressed without any ambiguity. Already on p. 19 of my paper of 1895* have I called attention to the fact that Bonnet's woodcut was a misrepresentation of my own views and have on Plate IV., Fig. 81, reproduced a hardly known figure of Kölliker's of the rabbit's blastocyst, which, on the contrary, is in complete accordance with those views. Misrepresentations, however, are hard to kill.

In Hertwig's 'Handbuch der vergl. Ent-

* *Verhandel. Kon. Akad. Amsterdam,* Vol. IV., No. 5.

wickelungslehre der Wirbelthiere' (Bd. I., S. 917) and in Weber's 'Säugethiere' (S. 284) similar transpositions of my intentions in insisting on the recognition of a distinct trophoblast have not crept in. I cite from Hertwig:

"Die verschiedene Entwickelung der Trophoblastes hat Hubrecht in folgenden Sätzen kurz zusammengefasst." "Die von mir Trophoblast genannte Kiemschicht ist für die Anheftung des Säugethierkeimes an die mütterlichen Gewebe in erster Linie bestimmt: dabei entwickeln sich zu gleicher Zeit in der mannigfaltigsten Weise lokalisierte oder über die ganze Oberfläche sich orstreckende Wucherungen, welche zur Ernährung des Embryos dienen." "Der definitive formative Epiblast welcher als sogenannte Keimscheibe oder Embryonalschild auf der oberen Fläche der Keimblase hervortritt, ist zur Zeit seines ersten Auftretens nie an der Oberfläche gelegen, sondern immer von Trophoblastzellen überlagert.

"Die Art und Weise wie diese Ueberlagerung des formativen Epiblastes durch Trophoblastzellen ein Ende nimmt, ist sehr verschieden; entweder entsteht zwischen Epiblast und Trophoblast ein persistierender Raum, welcher etwas später zur Amnionhöhle wird (*Erinaceus, Arvicola*), oder es tritt eine engere Verwachsung von den Epiblastrandern mit dem Trophoblast ein, worauf ein Durchbruch der deckenden Trophoblastzellen erfolgt, welche letztere später zu Grunde gehen (*Tupaja,* Maulwurf, vielleicht auch Fledermaus und Schwein) oder endlich, es wird die trophoblastische Deckschicht oberhalb der Keimscheibe sehr erheblich abgeflecht, wodurch der formative Epiblast und der Trophoblast dem Anschein nach in engstem genetischem Verbande stehen, während in Wirklichkeit der Verband zwischen dem peripheren Bezirk des Trophoblastes und seinem als Deckzellenschicht zu bezeichnenden Abschnitt auch hier die primäre, die anfänglich kontinuierliche Verbindungsweise gewesen ist (*Lepus, Sorex.*)" "Der Entwickelungsgang kann eine Abkürzung erfahren, indem die Amnionhöhle innerhalb eines vom Trophoblast verfrüht abgetrennten Epiblastzellenkörpers spontan erscheint (*Cavia, Pteropus*)."

I hope that these quotations may dispel eventual doubts about the significance of the term trophoblast.

I have now to consider the application of that term in gynæcology, more especially its application to the placentation of man and of other mammals. I find that its use has become more general with English and American than with German authors, and at the same time it would seem as if also in this case a tendency to misunderstand the real significance of the term has sprung up.

This tendency is very naturally explained when we consider how important a factor for the process of placentation the proliferation of trophoblast cells has been shown to be. This proliferation is especially very luxurious in man (Siegenbeek van Heukelom, Peters) in the hedgehog (Hubrecht), whereas in many other insectivora and rodents its multiform complications are exceedingly varied. In consequence many investigators have been led to believe that the name trophoblast was originally restricted to those proliferating regions, whereas we have demonstrated above that this has never been so. Add to this that the proliferation of cells contributing to the formation of the placenta yet in another way is apt to lead to confusion because such proliferation is in no way limited to embryonic cells, but is also noticed—sometimes to a more considerable extent—in maternal cells belonging to the epithelial lining of the uterus or to the subepithelial maternal tissues. In this way debates have arisen between Strahl and

Lüsebrink on one side and Duval, van Beneden and myself on the other, of which debates the object was to make out in how far the material of the proliferating placenta should be looked upon as maternal and in how far as embryonic, trophoblastic tissue.

These debates will no doubt, in the course of time, as the number of carefully observed cases increases, lead to a unanimous interpretation. As it was, I have myself, for sheer diffidence of attributing too prominent a part to trophoblastic proliferation (of which I was, nevertheless, together with Duval, the first advocate), in one case stopped short of the real solution, and have for the hedgehog restricted the extent of the trophoblast more than was necessary. Since then I have corrected this in a doctor dissertation of one of my pupils (Resink, 1903), but there is no doubt that I am myself thus responsible for a certain amount of vagueness and misrepresentation which has prevailed in the application of the term trophoblast to different placentas, more particularly of man and the monkeys, where the question arose in how far certain syncytial tissues should be looked upon as maternal or as embryonic. Even for pathological anatomy this proved to be a momentous question in so far as the deciduoma malignum, if traced to remains of trophoblast cells, would be very different from other deciduomæ, that found their origin in maternal tissue.

Now that the placentation of *Tarsius, Tupaja, Sorex, Vespertilio, Cercocebus, Talpa, Galeopithecus, Sciurus, Lepus*, a. o., has been more carefully examined (trophoblastic proliferation having been figured by Selenka as early as 1887, for one of the Didelphia [*Opossum*]), divergence of opinion will in a few years hence have been replaced by unanimity also on this head.

And then the application of the name trophoblast to those placental elements that arise from the embryonic layer originally designated by that name will be in no way confusing, but will, on the contrary, contribute to keep before our eyes the intimate relation between the facts as they take place under our eyes and their phylogenetic origin.

With perfect justification Strahl has protested (Hertwig's 'Handbuch der Entwickelungslehre der Wirbelthiere,' I., 2, p. 311) against a misapplication of the terminology, which I have attempted once more to explain in this article, when authors who have insufficiently studied the subject even go so far as to speak of a *maternal* trophoblast beside the embryonic!

I hope that this paper may henceforth render misinterpretations such as are discussed in it impossible.

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SCIENTIFIC BOOKS.

Geographic Influences in American History. By Albert Perry Brigham, Professor of Geology in Colgate University. Boston, Ginn & Co. 1903. Pp. 365.

Professor Albert Perry Brigham, of Colgate University, has made a notable contribution to American geographical literature in his book, 'Geographic Influences in American History,' published by Ginn and Co. The divisions of the book are mainly physiographic, but the author has not allowed this subject undue prominence.

Chapter one is entitled 'The Eastern Gateway of the United States.' The central idea is the development and importance of the Hudson-Mohawk valley—its physiographic origin and its influence upon American history. Its importance in the Revolution and the war of 1812, and the successive waves of immigration which passed through it and left their record in the 'successive layers of geographic names' are well brought out. The interdependence of the valley and the cities which have grown up along it and the conditions of growth of the metropolis at its mouth are discussed.

'Shore Line and Hill Top in New England' is the title of chapter two. A mountain foundation covered with drift is described as the physiographic type of the division. The influence of this structure upon life, upon agriculture and the diversion of the streams, thus causing the waterfalls so prolific of power, the resulting coast line and its result upon commerce are among the topics taken up. The history of New England, which has been directed to some extent by surface and climate, is discussed, but the author does not lose his balance. 'That environment influences character,' he says in summing up the influences of climate and physical features, 'need not be asserted; but one can not be sure in weighing its influences.' The coloring of the literature of New England by geographic influences is shown by selections from Emerson, Whittier and Thoreau.

The 'Appalachian Barrier' is discussed in the third chapter. The region of mountains, plateau and valley which stretches as a unit from New York to Alabama, together with its fringe of Piedmont Plateau and coastal plain, is described in popular language. The unity that such a barrier enforced upon the colonists and its effects upon the Revolution are the principal themes of the chapter. The development of routes across this barrier, first by trails and later by railroads, is described.

Chapter four has the title, 'The Great Lakes and American Commerce.' The Great Lakes, their probable origin, their geographical features and their historical and industrial influences are prominent topics. Duluth, Milwaukee, Chicago, Cleveland and Buffalo are accounted for on geographic principles. Niagara, past, present and future, is discussed. In discussing this great continental waterway from Buffalo to Duluth the fact, perhaps not generally appreciated, is emphasized that more tonnage passes through the 'Soo' than through the Suez canal, and that the commerce of Cleveland at times rivals that of Liverpool.

Chapter five deals with the prairie states, taking Illinois and Iowa as types. The introductory pages describe the visit of the Spaniard on the south and the invasion of the French trapper and trader on the north. In following the latter an idea is given of the drainage systems which provided paths for them. The topography of Ohio, Indiana and Illinois is contrasted with that of New England. Both are glaciated regions, but the former has received the ice action on a fairly level surface, so that the glacial detritus is more evenly spread, the finer materials of ice action have not been swept away and the level, fertile prairies are the result. The author takes up the settlement of this district, the routes of immigration, the development of its social and financial problems and the location and growth of its cities.

'Cotton, rice and cane' is the title of the chapter which deals with the group of southern states. The coastal plain with its level surface and sluggish tidal rivers is described as the typical structure of most of the states of this division. Virginia, North Carolina, Georgia and Alabama have in addition considerable area in the Piedmont Plateau, while Florida has a structure rather peculiar to itself. In South Carolina three zones of life and industry are described, each a good example of 'earth control.' Sea Island cotton, rice and truck farming are characteristic of the coast lowlands and islands. Here is the 'black belt.' Between this and the plateau is a belt of sandy country whose inhabitants, the 'sand hillers,' share the poverty of the soil. In the northwestern corner of the state is the plateau country whose poorer soil and cooler climate made slavery less profitable, and the whites are more numerous than the colored people.

North Carolina, while rich in land and with a genial climate, drew her population from Virginia and South Carolina. The sand reefs on the coast and lack of harbors cut her off from the sea. Florida is described as altogether coastal plain, and the most youthful of the states. The lack of water power and coal and the scarcity of most minerals would seem to justify the author's prophecy that the country will not give way to the town to the same extent as in most states.

The evolution of the plantation, the growth of the slave system and the present social and

industrial development are discussed from the standpoint of geographic principles.

The civil war is taken up from the author's standpoint in chapter seven. The rivers, mountains and topographic features that had a bearing on the conduct of the war are first considered. Then follows an analysis of the geographic influences upon the three great campaigns of the war: the opening of the Mississippi, the operations around Chattanooga and the Virginia campaign. Although, perhaps, a digression from the general plan of the book, this chapter is exceedingly interesting and indicates the possibilities of a line of research which has been strangely neglected by historians.

The arid regions of the west are described in chapter eight. After a discussion of the varied physiography of these regions and the causes which deny to them their share of rain, the author presents an interesting example of geographical politics.

During the early part of the decade from 1885 to 1895 there was a slight increase of rain in this region which coincided with the rush of settlers into western Kansas and Nebraska. Later the normal dry seasons followed and brought disaster. The people, pressed for ready money, accepted populistic doctrines and the movement grew out of 'the failure of a frontier population properly to adjust itself to geographic conditions.'

The problems of irrigation constitute a considerable portion of this and the succeeding chapter. The origin of irrigation, its possible effect upon society, its intensive agriculture, its present status and its regulation and distribution are well brought out. One leaves the subject with a lively impression of this problem which most of us not immediately concerned do not fully appreciate.

Chapter nine treats of the mountain and plateau states under the title, 'Mountain, Mine and Forest.' Colorado is described as the typical western state. After briefly treating of the physiography, the mining interests are taken up in some detail. The origin of veins and lodes, the influences of mine development upon society and the location and growth of Denver, Pueblo, Leadville and other towns are described. After describing their physiography, the problem of irrigation in Arizona, Wyoming, Idaho and Nebraska is discussed. The influence and preservation of forest reserves in these states are taken up.

'From the Golden Gate to Puget Sound' is the title of chapter ten. The three Pacific states are described as having a common backbone in the Sierras, a similar depression to the west of these and a common range fronting the sea. The geological development which produced the present surface and made possible the mineral wealth is discussed in a readable manner. To this land the Spaniard came, but his occupation gave only a 'sentimental background' to its history. The Mexican occupation left scarcely more impression; the Yankee traders bought their hides, took them to New England, carried them back and sold them as shoes in California ports. The author sketches the discovery of gold, the rush of the 'forty-niners' and the subsequent steps to the admission of California as a state. Coming down to 1900, it is interesting and significant to learn that the agricultural output of the state exceeds over threefold the value of its mineral product. The location of cities and the relation of the Isthmian Canal to the Pacific coast are among the topics considered.

Oregon and Washington are discussed together in history, topography and products. Their settlement against the efforts of the Hudson Bay Company is sketched. Their coal fields and forests are described and their commercial possibilties are prophesied in these words: 'He need be no seer who sees cities like those of the Atlantic standing under the western sun.'

The eleventh chapter deals with 'Geography and American Destiny.' The author finds in American history one geographic fact of overshadowing importance: 'A wide ocean separated an advanced civilization and a relatively dense population from a wide, rich and almost unoccupied continent.' The growth of the nation by conquest and purchase, its development along geographic lines and its advent into more active world relations in 1898 are described. The dominant topic of the chap-

ter is American unity. Intercommunication, common business interests and common national ideals are named as factors contributing to this unity.

The final chapter treats of 'Governmental Study of our Domain.' It contains a description of the work of the Geological Survey, Coast Survey, Fish Commission, Department of Agriculture and other departments of governmental activity.

'Geographic Influences' is almost a pioneer in its line. Among the geographic influences the author places the greatest emphasis upon physiography, but the treatment is not overbalanced. A quality which can hardly be shown in a review is the vigorous, attractive style. The author presents a wealth of facts without a trace of 'statistical' manner. The illustrations are well selected and several maps are included in the work. The book should have a good circulation among those readers who are interested in American history. It will also appeal strongly to an increasing number of students who are considering the relations of the earth sciences to the politico-historical sciences.

F. V. EMERSON.

CORNELL UNIVERSITY.

DISCUSSION AND CORRESPONDENCE.

THE METRIC SYSTEM.

TO THE EDITOR OF SCIENCE: I feel that I must congratulate your readers on the review of 'The Metric Fallacy,' published in your issue for August 5, from the pen of Professor W. Le Conte Stevens. Passing by the charges of partisanship and intolerance for the moment, as of no real importance, the review exhibits a comprehension of the subject which is rare—I might almost say unique—among metric advocates. My purpose in writing this letter is to point out that Professor Stevens's admissions are of far greater importance than he seems to suspect, and I trust you will find a place for it—not as a reply to the review, nor as a defence of the book, but as a continuation of an important discussion.

As I have pointed out in a special chapter of 'The Metric Fallacy' (The Pro Metric Argument), the metric advocates have based their case upon the belief that the change to their system is an easy one. This belief is the chief burden of the pro-metric statements made before the house committee on coinage, weights and measures, of the fifty-seventh congress, and it was largely through its reiteration that the favorable report of that committee was obtained. In the above-named chapter will be found quotations from the statements made by a dozen metric advocates before that committee in which the longest period named for the change is five years. I can, however, ask you for space to repeat but one of these—the gem of the whole collection—which came from Lord Kelvin (italics mine):

> I believe that *in a fortnight* people would become so accustomed to the perfect simplicity and easy working of the metrical system, that they will feel that instead of its being a labor to pass from one system to the other, *it will be less than no labor.*

This opinion, I should add, was repeated with approval at the great discussion of the American Society of Mechanical Engineers in 1902.

Through a clipping bureau, I have received from Great Britain hundreds of newspaper clippings pertaining to this agitation in that country, and the case there, as here, is based upon this assumption.

It would, indeed, be superfluous to mention this contention of the metric advocates, except to point out that Professor Stevens is so far in advance of his associates as to frankly tell us that 'reasonably complete assimilation will take several generations' and that 'none of us of to-day will live to see anything better than good progress on the part of the general public in getting accustomed to the new standards and in losing devotion to the old ones.'

Among those whose opinions as to the shortness of the transition period are given in the chapter on 'The Pro Metric Argument,' are: Elihu Thompson, Harvey W. Wiley, S. W. Stratton, Simon Newcomb and Lord Kelvin. Professor Stevens seems to think I have not treated these opinions with due respect, but in view of the above quotations from his review it is hard to see wherein he respects them more than I do. He plainly regards

them as worthless and I can go no farther than that.

In view of the facts regarding the use of old units in all metric countries, how is it possible to treat with respect the repeated assertion of the metric advocates before the house committee on coinage, weights and measures that the change with us will require but from two to five years, and what matters it that these statements are made by men of distinction, chief of whom is Lord Kelvin?

Following the assumption of the ease of the change, and a necessary outgrowth of it, is the assumption that it has been made in countries in which the system has been adopted. Thus in 'The Coming of the Kilogram,' published by the English Decimal Association, and thus made official, we may find:

'There are now no longer a great number of sets of weights and measures in use among the civilized peoples of Europe; there are really for all useful purposes two only,' these being, of course, the English and the metric.

It would again be superfluous to mention this contention of the metric party, but for Professor Stevens's remarkable admission:

> A century has not been sufficient to cause the abolition of old names and units among the common people of France, resort to them being usual when no penalty is involved. The same is true in Germany and Switzerland and in every other country where the metric system is, in business transactions, either obligatory or permissive.

If it is the common experience that the people of metric countries resort to the old units when no penalty is involved, then it is clearly impossible to bring the system into common use in this country where general compulsion is admittedly impossible. Furthermore, if this system is so superior, how can this admission be true? Why should it be necessary to compel people to use such a wonderfully superior thing as the metric system is represented to be? It is certainly the only case of the kind on earth.

Just as the pro-metric case has been based upon the belief that the change will be short and easy, so the anti-metric case has been based upon the belief that, as shown by the experience of other countries, it would be long, difficult and costly, and that the long transition period would be one of great confusion. While I have made no count of its pages, it is safe to say that eighty per cent. of 'The Metric Fallacy' is devoted to enforcing this and to nothing else. Professor Stevens now comes to our assistance, however, and admits that a century of compulsory laws in France, and thirty years of them in Germany, have been insufficient to complete the change, that 'certain people would lose money and otherwise suffer much inconvenience from the change,' that 'all possible consideration should be accorded to those whose large pecuniary interests are affected' and that 'the first result will be not the abolition of confusion, but the increase of confusion,' and how long he expects this period of added confusion to be is shown by his admission that 'none of us of to-day will live to see anything better than good progress on the part of the general public in getting accustomed to the new standards and in losing devotion to the old ones.'

It thus seems to me that Professor Stevens has admitted pretty much all that we have contended for. There remain, in fact, but two important differences between us—he regards the change as worth this cost and this period of confusion, while Mr. Dale and I do not, and he regards the change as feasible, while, in a manufacturing sense, we regard it as impossible.

In common with other scientific men, Professor Stevens fails to recognize the root of the difficulty and of the opposition, the difficulty of changing established manufacturing standards, of which textile, screw thread and pipe standards are representative. I am, in fact, in this respect, disappointed in this review, as it ignores what I regard as the two most important chapters of my portion of the book 'Scientific and Industrial Measurements' and 'Scientific and Industrial Difficulties.' I have there pointed out—I believe for the first time and without reflection upon either party—the reason why scientific men favor the system while manufacturers oppose it. In brief, this is because 'the scientific use of measurements consists in measuring existing things; the industrial use of measurements

consists in making things to required sizes.' In scientific work the change involves a change in measuring instruments only, while in industrial work it involves a complete change in standardized manufactured goods—a change which manufacturers know to be impossible. I can not ask for space to enlarge upon this, but it explains completely the different attitudes of the two parties.

Mr. Dale and I profess to speak as those who are experienced in manufacturing, and as having given protracted study to the effect of this change upon manufacturing industry. As scientific men have, rightly, no respect for the opinions of the non-scientific upon scientific questions, so we have no respect for the opinions of those who have no expert knowledge of manufacturing upon the effect of this change upon manufacturing industry. We acknowledge that our language is harsh, but it is in no way more so than the stock dismissal of all objections to this change as due to 'ignorant prejudice,' and it is not for those who have always treated our views with contempt to object when they find their own non-expert views treated in the same manner. While Professor Stevens, although opposed to us, has been so generous as to frankly give recognition to our book by saying that it contains 'a mass of information that must be taken into account,' he should, and doubtless does, know that the chief weapons of the prometric party have been ridicule and contempt, and he should not criticize us for making use of the same guns. We believe that we are justified in using them when discussing opinions that entirely fail to recognize the importance, in many cases the existence, and in all the formidable nature of the difficulties that surround the manufacturing side of the question. What scientific men need to learn more than all else in connection with this subject is that their experience, their knowledge and their horizon do not include manufacturing. They have uniformly failed to recognize the difference, or, indeed, that there is a difference between measuring things and making things. They are measurers, not makers, and their opinions have no value and no application as related to manufacturing.

The proposition is that we make this change in industry and commerce. It is, therefore, an industrial and commercial, and not a scientific question. It is the province of scientific men to determine the weights and measures which they shall use, but when they endeavor to foist this thing upon others who must pay the cost, while they pay nothing, as they have succeeded in doing wherever the system has been adopted, and as they have tried to do here through the hearings of the house committee on coinage, weights and measures, they simply meddle with other people's affairs, and exhibit an assurance which furnishes the occasion, and I believe the justifiable occasion, for the vigorous language of 'The Metric Fallacy' which, however, has obviously not prevented Professor Stevens from learning some wholesome facts from it.

Contrast the enormous development of organized manufacturing to-day with its comparatively trifling development at the introduction of the metric system in France a century ago. Even with that trifling development Professor Stevens admits that 'conservatism has been too strong and vested interests too great to permit the enforcement of any interfering laws.' Is it not obvious that the change in France under the conditions of a century ago was easy compared with what it now is here? If, starting with the conditions of 1793, and after a century of compulsory laws, resort to the old units is in France still 'usual when no penalty is involved,' how many centuries must elapse before that resort becomes unusual under existing manufacturing development here where general compulsion is not to be contemplated?

Were the metric party to unite on the admissions made by Professor Stevens when presenting their case to the committee on coinage, weights and measures—and I shall see to it that they go before any such future committee—the metric bill would be killed at the hands of its friends as it was last winter killed at the hands of its enemies, for, contrary apparently to the impressions of Professor Stevens, the metric system from a legislative standpoint is the 'deadest' thing in this country.

F. A. HALSEY.

NEW YORK, August 30, 1904.

THE SONG OF BIRDS.

In view of the interest attached to the experiments made by Professor W. E. D. Scott with a view to ascertaining whether the song of birds is instinctive or imitative it may be well to recall that very similar experiments were made over a century ago. Wallace notes in his essays on 'Natural Selection, etc.,' that "The Hon. Daines Barrington was of the opinion that 'notes in birds are no more innate than language is in man, and depend entirely on the master under which they are bred, *as far as their organs will enable them to imitate* the sounds which they have frequent opportunities of hearing." An account of his experiments is given in the *Philosophical Transactions* for 1773, Vol. LXII., and his results were practically the same as those obtained by Professor Scott, the young birds acquiring the song of their foster parents and not the notes of their own species. Barrington notes that the birds must be taken from the nest when very young, as they are apt pupils and learn the call notes of their parents at an early age.

F. A. L.

Museum of the Brooklyn Institute,
August 28, 1904.

DOCTORATES AND FELLOWSHIPS.

To the Editor of Science: In Science for August 19 is the remark: 'It is somewhat surprising that California and Stanford have together conferred the degree (of Ph.D.) but twenty-five times in seven years.' Among the reasons for this are two: the authorities of this institution advise their advanced students, for the sake of breadth of experience, to take a part of their work in the east or in Europe. This is generally the concluding part. On the other hand, the California institutions grant few artificial aids to students as such. At Stanford this is regarded as a matter of principle, tuition being free to graduate students. By the wider introduction of the 'trading stamp system' in higher education, the number of degrees could be greatly increased, but with no gain to science or art.

David Starr Jordan.

SPECIAL ARTICLES.

LIMITATIONS OF THE KLINOSTAT AS AN INSTRUMENT FOR SCIENTIFIC RESEARCH.

The klinostat is an instrument used mostly in plant and animal physiology, revolving about either a vertical or a horizontal axis, for the purpose of neutralizing the tropic (curving) effects of light and gravitation. It was first made practical and introduced into laboratory work by Sachs in 1872, and fully described with additional improvements in several papers in that memorable series of pioneer research entitled 'Arbeiten des botanischen Instituts in Würzburg.'

The instrument as commonly manufactured is actuated by springs, and may be given several speeds, usually one revolution in ten, fifteen, twenty or thirty minutes. In the use of the instrument for demonstration and research during the past thirty years, it has generally been thought immaterial to consider the angles at which plants are secured to the horizontal klinostat for the purpose of neutralizing the tropic effect of gravitation, or to consider the distribution of light when plants are revolved on the vertical klinostat for the purpose of neutralizing the tropic effect of light. In addition to the foregoing relations of the instrument to gravitation and light, the present tendency to substitute electric and water motors for springs to drive the machines introduces relations of speed. The research of the past ten years has indicated very important limitations to the use of the klinostat, and the most general of these are pointed out in the following lines, and should be recognized by every one using this instrument.

Evenness of Speed.—The first requisite for securing evenness of revolution is the centering of the load. An excentric load will give a slower speed periodically, and this will bring curves. It has been shown in my own laboratory that, in a speed of one revolution in ten minutes, a periodic retardation of but a second or two, due to excentricity of load, will cause, after a long interval, a geotropic curve by the summation of stimuli.

Avoidance of Centrifugal Effect.—It is well known that in rapid revolution, plants simulate toward the so-called centrifugal force

their behavior toward gravitation. The revolution must be so slow therefore as not to bring in the centrifugal effect. Czapek* has measured the sensitiveness of plants to centrifugal force when gravitation is neutralized, finding that by sufficiently long continuance of the experiment, curves will come when the rate of revolution gives an acceleration exceeding one thousandth part of the acceleration of gravitation (.001 g.), but do not appear, at least for many hours, at a lower rate of speed. The relation of the acceleration due to rotation to the acceleration of gravitation is found by the formula $F = 4.024\, r/t^2$, in which r is the distance from the center of rotation in meters, t is the time of one revolution in seconds, and F is the ratio of the acceleration of rotation to the acceleration of gravitation. By applying the formula it will be found that an object rotating once in twenty seconds, at a distance of 10 cm. from the center will have an acceleration approximately equal to .001 g. Such a rate of revolution would, therefore, be within the limit of safety for objects less than 10 cm. from the center of rotation. To make the results doubly sure, one could keep a speed not to exceed one revolution per minute.

Avoidance of too Slow Revolution.—Not only is there an upper limit of speed, but a lower limit also when one wishes to neutralize the tropic effect of light or gravitation. A revolution that is so slow as to allow a plant to remain in one quadrant of its path, or, in certain cases, in one semicircle, for a period equal to the period required to produce a curvature when the plant is at rest, will produce a curvature of the plant on the klinostat, in which the apex of the plant member will, by combining its curves during a complete revolution of the klinostat, describe a line, as it moves back and forth, or a circle as it circumnutates, the one or the other according to the position of the plant with reference to the axis of revolution of the klinostat. Indeed, the recent work of F. Darwin and Pertz† may be taken to indicate that a plant on the horizontal klinostat, remaining in a quadrant for a period even less than the latent period of curvature may keep up a rhythmic oscillation as it revolves. Since at a temperature of 20° C. the latent period for curvature of some roots and stems is not over twenty minutes, and the perception period of some plants for gravitation at 20° C. is less than fifteen minutes, safety would advise that thirty minutes should be the limit for slow revolution, the plant then being in each quadrant but seven and one half minutes.

Not All Tropic Influences can be Neutralized by the Klinostat.—It is often desirable to revolve a plant on the klinostat, thereby freeing it from the tropic effect of gravitation in order that some property of the plant, ordinarily veiled by the control of gravitation, may be brought to expression. Thus the bending of several roots from light has been discovered by the use of the klinostat, the same roots showing no response to light as long as the stronger influence of gravitation was operative. It is well to know, however, that the so-called *autotropism* of plants is uninfluenced by the klinostat, and that this tendency of plants to grow straight must always operate against the formation of curves in plants that might otherwise show responses. This autotropism of roots and stems is of no inconsiderable moment, since curves of several hours' formation are by it completely obliterated while the plant is so revolved on the klinostat as to neutralize the tropic effect of gravitation.

It is of the first importance to know that only those parts of plants which are physiologically radially symmetrical will grow straight while revolving on the klinostat with horizontal axis.* A plant may be radially unsymmetrical toward gravitation, in which case, revolution on the klinostat will not release it wholly from the tropic influence of gravitation; or it may be radially unsymmetrical physiologically because of unsymmetrical behavior due to internal stimuli. In the revolution of the latter kind of plant member on the klinostat (many leaves), we may have curves due to unequal growth on the one side or the

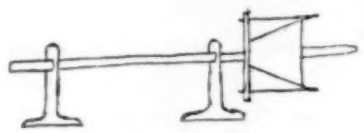
* *Jahrb. wiss. Bot.*, XXVII., 1895, 243.

† *Annals of Botany*, XVII., 1903, 93.

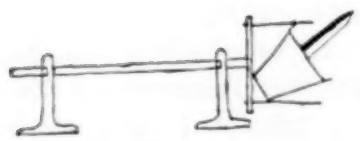
* Noll, *Jahrb. wiss. Bot.*, XXXIV., 1900, 459.

other, this unequal growth being known as *epinasty* or *hyponasty*.

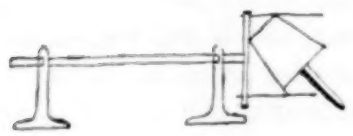
How May Radially Symmetrical Plant Members be Placed on the Klinostat to Neutralize the Tropic Effect of Gravitation?—For neutralizing the tropic effect of gravitation the horizontal klinostat is always used. There are three positions the plants may occupy as they are fastened to the klinostat. The positions are illustrated by the four diagrammatic figures *A*, *B*, *C* and *D*. The position in *A*, in



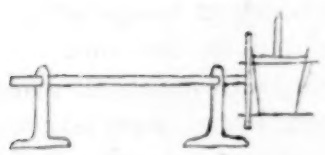
A



B



C



D

which the axis of the plant is made parallel with the axis of rotation, is the one which has been most used in physiological work. The position in *B* and *C* is one in which plants have seldom been designedly placed, but in which parts of plants have been generally left without concern if in any way they came into this position. The question now to be considered is this: Are orthotropic parts of plants revolving on the klinostat as indicated in *A* and *B* freed from the tropic effect of gravitation? A consideration of the following results of recent investigation answers this question decisively in the negative. Czapek* found some evidence to show that primary roots, and Miss Pertz† showed that grass stems are not equally stimulated by gravitation at equal distances above and below the horizontal plane. Work done in the botanical laboratory here by Miss Haynes and myself, and soon to be published, demonstrates that both orthotropic roots and stems receive their strongest gravitation simulation for bending when those plant members are placed in the horizontal position; and that stems are more strongly stimulated to bend when inclined below the horizontal than at an equal angle above the horizontal, while roots are more strongly influenced when inclined with their tips above the horizontal than at equal angles below the horizontal. A stem, therefore, in the position shown in Fig. *B* does not feel the stimulation of gravitation so effectively as in the position 180° removed as in Fig. *C*. It is, however, entirely conceivable that the foregoing relations may be true for plants at rest, but may not hold for plants rotating on the klinostat. But numerous tests made by myself with several species show that both roots and stems rotated on the klinostat in the position shown in Figs. *B* and *C*, or in any oblique position between the horizontal and vertical, bend toward the horizontal as the experiment progresses. Not only then is the oblique inclination of the plant axis during revolution a faulty one, but the horizontal position as in Fig. *A* is not proper. In the latter position, it is true, the plant will grow straight, but it will grow straight for the same reason that it grows straight when at rest in the vertical position—because with each deviation from the horizontal position its geotropic response will send it back to the horizontal.

If one wishes to rotate orthotropic stems and roots on the klinostat so as to free them from the tropic effect of gravitation there is only one position possible, and that is the vertical position, by which the plant axis dur-

* *Jahrb. wiss. Bot.*, XXVII., 1895, 283.

† *Annals of Bot.*, XIII., 1899, 620.

ing revolution describes a vertical plane as shown in Fig. *D*.

If one wishes while the plant on the klinostat is freed from the tropic effect of gravitation to apply some stimulus for the study of a tropic reaction, the stimulus should be employed so as to cause curves in the plane of revolution; for as soon as the tip of stem or root deviates from the vertical plane, gravitation seizes upon it to turn it to the horizontal.

Neutralization of the Tropic Effect of Light.—Experiments in which plants are rotated to neutralize the tropic effects of gravitation have been mostly carried on in the dark-room with seedlings. Such conditions and material are easily obtained and managed. Sometimes, however, when the experiments are prolonged, and older plants are used, it is necessary to do the work in the light to avoid the appearance of pathological conditions. The requirements for such work are as manifold as the end sought and can not be considered here. It may be indicated that when the experimental plants are subjected to one-sided illumination as before a window, there are several adverse conditions that are not easily overcome. The common statement that the plants are to be rotated with the axis of the klinostat parallel with the window in order to neutralize the tropic effects of light leads to humiliation or self-deception. If the part of the plant sensitive to light is not at the center of rotation, it alternately as it revolves approaches and recedes from the light, receiving therefore more light on one side than on the other, and responding with phototropic curves. Or if so placed that it does not approach and recede from the window, it may on the horizontal klinostat, receive on one side its principal illumination from the sky and, on the opposite side of the plant, from the earth, thus again forming phototropic curves because of unequal stimuli. Several plants in a pot revolving on a klinostat with axis parallel with the window are likely to show curves because of the shadows which the plants cast on one another. Finally, the effort to neutralize the tropic effect of both light and gravitation on the same plants at the same time is so difficult as to require very special attention to the management of the light, unless the work is done in a plant-house with full exposure. A klinostat, such as has been devised, giving both vertical and horizontal revolution at the same time is not a guarantee against faulty results.

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QUOTATIONS.

THE PRESIDENTIAL ADDRESS BEFORE THE BRITISH ASSOCIATION.

MR. BALFOUR'S presidential address to the British Association naturally recalls another occasion in the history of that body when the chair was filled, not exactly by a prime minister, but by one who had been and very shortly again became prime minister. There is a marked resemblance between the address delivered at Oxford in 1894 by the late Lord Salisbury and that delivered yesterday in the seat of the sister University by his nephew and successor in office. It is entirely right and fitting that this should be the case, because on both occasions the British Association has listened, not to an expert in any one of the sciences, but to a student of the true *scientia scientiarum*. In a limited sense that proud title may be bestowed, as it was yesterday by Mr. Balfour, upon the science of physics. But the real *scientia scientiarum* consists in the assimilation by men of broad and sound general culture of the best and most assured results of the labors of specialists in many fields of research, and in the application of these results to life and conduct in domestic affairs, and to the solution of imperial problems in the wider arena of statesmanship. Of the countless millions who own allegiance to the king there is no man so deeply bound by duty and responsibility to master that difficult science as the prime minister of Great Britain. The obligation, it must be confessed, has sat very lightly upon many holders of that high office. We would fain hope that the appearance of two prime ministers in succession, showing incontestably that they possess a genuine appreciation of the vital necessity for knowledge in the direction of affairs, may be taken as a sign that a higher standard than that of political dexterity in appealing to ignorance

may by degrees be firmly established. We may anticipate the sneer that speculations on the nature of matter do not help much in dealing with what is called 'practical politics,' apparently because it consists chiefly of unpractical and impracticable verbiage. Perhaps not; but the intellectual habit and the intellectual capacity which impel and enable a man immersed in work to keep himself acquainted with the changing aspects of science upon which such speculations depend, do most potently help him in whatever business it may be his lot to transact. The precise color of his speculations is not very material, since it depends upon personal idiosyncracies of which the man himself could give none but the most inadequate account. In essence these speculations are as old as human thought, and all through the ages we find the most powerful intellects ranged on opposite sides. In form the speculations are constantly changing, and so are the names of the opposing parties. What we have to ask about a man is not on which side he stands, but what mastery he shows of the contemporary scientific achievement which gives to the secular controversy the form it wears for the men of his own time. That mastery, according to its degree, makes him an efficient intellectual force in whatever field he may choose to exert himself.—London *Times*.

AN ANCIENT FICTION.

The astonishing longevity of popular delusions in natural history is nowhere better illustrated than by the credence given even by educated folk at the present day to alleged discoveries of frogs, toads and other animals within hermetically sealed cavities, such as inside overgrown hollows of trees or closed interior spaces of rock.

Dr. Traquair, director of the Edinburgh Museum, mentions a letter published in the London *Times* a few years ago by the late Miss Amelia B. Edwards, the eminent Egyptologist, on the finding of a live toad deep down in boulder clay at Greenock, and refers to another accomplished author who took umbrage because some one ventured to question his assertion that live frogs occur in the Old Red Sandstone. Dr. Traquair is right in supposing beliefs of this nature to have considerable antiquity, although he professes ignorance as to how far back they can be traced into the past.

Probably not many will be surprised to learn that this popular delusion has a continuous history in literature of at least four hundred years; and if we include the so-called 'subterranean fish' of Narbonne, to which a special chapter is devoted in Rondelet's 'Ichthyology' (1554), a form of it is traceable as far back as the time of Aristotle. The occurrence of live fish underground, and the singular mode of taking them with the spade instead of the net, or hook and line, is mentioned repeatedly by classic authors, the most particular account being that of Polybius ('History,' xxxiv.). Older than these, though of scarcely germane nature, are the Biblical and secular legends of miraculous suspension of vital functions, of which the tale of the Seven Sleepers is an example.

References might be given to more than a score of sixteenth to eighteenth century writers who make more or less particular mention of the occurrence of live animals in cavities long closed to the air, the list including such prominent names as Conrad Gesner,* Agricola,† Francis Bacon,‡ Athanasius Kircher,§ Libavius‖ and Astruc.¶ It is not, however, worth while to take further notice of these early accounts, unless it be one which is remarkable for its *naïveté* and evident truthfulness. This narrative is to be found in the surgical works (book xxv., cap. 18) of Ambroise Paré, court physician of Henry III.,

* 'De omni rerum fossilium, etc.,' Zurich, 1565.

† 'De Animantibus subterraneis,' Cap. XXXV., Wittenberg, 1614.

‡ 'Natural History,' opus posthumum, cent. vi., § 570. Bacon says here: 'The ancients have affirmed that there are some herbs that grow out of stone; which may be, for that it is certain that toads have been found in the middle of freestone.'

§ 'Mundus subterraneus,' lib. viii., Amsterdam, 1664 and 1678.

‖ 'Singularium,' part i., de carne fossile; part iv., de Batrachiis, cap. 25.

¶ 'Mémoires pour l'Histoire naturelle de la Province de Languedoc,' part iii., chap. 10, Paris, 1737.

king of France and Poland, and sometimes called 'father of French surgery.' We would fain reproduce it in all the quaintness of the original, as follows:

Étant à une mienne vigne, près le village de Meudon, où je faisois rompre de bien grandes et grosses pierres solides, on trouva au milieu de l'une d'icelles un gros crapaud vif, et n'y avoit aucune apparence d'ouverture, et m'esmerveillai, comme cet animal avoit pû naître, croître et avoir vie. Lors le Carrier me dit qu'il ne s'en falloit esmerveiller, parce que plusieurs fois il avoit trouvé de tels et autres animaux au profond des pierres, sans apparence d'aucune ouverture.—(1564).

Differing in nowise from the above, except that they are attested by numerous reputable witnesses, are the accounts presented to the French Academy in the early days of its history, one such being preserved in the *Mémoires* for 1719, and another in the volume for 1731. Nor should one fail to note what the old Toulonese historians have said concerning edible mussels which the inhabitants cracked out of the solid rock in the harbor of their city. Instead of going clam-digging, those in search of a delicacy sallied forth with a hammer, and the shell-fish so obtained were said to have had an admirable flavor.*

Coming down to our own time and country, the curious will find in the *American Journal of Science* for 1822 (Vol. XIX., p. 167) an edifying description by Mr. David Thomas of the discovery of a live toad in a block of limestone during the excavation of the Erie Canal. The details of this incident, even including the fact that the toad got away, are authenticated by various 'witnesses of unimpeachable veracity,' one of them afterwards becoming a state senator, as we are told. And so with similar anecdotes, which meet the eye from time to time in the daily newspapers.

Not the slightest suspicion of untruthfulness in any of these instances is attached either to the narrator or to the spectators who corroborate the original statement to the best of their belief and knowledge. It is probably furthest from the minds of these worthy people wilfully to deceive the public, and yet, on the other hand, a little reflection shows that from the very nature of things such tales are incredible, and those who vouch for them must be mistaken in their observations, even as the most alert and sharp-sighted persons are deceived by the feats of a prestidigitator. The evidence given by the spectators would, so far as appearances go, be perfectly trustworthy, but when we subject their accuracy of observation and memory to the test of reason, which asks whether they relate that which is perforce impossible, the credibility of these witnesses is at once challenged. The fact remains that the sea serpent has not yet been captured, and in every instance these long-imprisoned frogs and toads and other animals have suddenly become vivified and succeeded in making their escape. Hence we are bound to agree with Dr. Traquair when he remarks that he 'should certainly not like to see the issue of a trial, civil or criminal, depend on the question of whether a man had seen the sea serpent, or had witnessed a family of young adders creeping down their mother's throat.'

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* Bouche, H., 'Chorographie, ou Description de la Provence,' vol. i., p. 924, Paris, 1664. The account evidently refers to boring mollusks.

SCIENTIFIC NOTES AND NEWS.

THE medal of the Society of Chemical Industry, awarded every second year for services to applied chemistry, has been presented to Dr. Ira Remsen, president of the Johns Hopkins University.

DR. LUDWIG SYLOW, professor of mathematics in the University of Christiania, has been made a foreign knight of the Prussian order 'pour le mérite.'

DR. MORITZ CANTOR, the well-known mathematician of the University of Heidelberg, celebrated recently his seventy-fifth birthday.

DR. WILLIAM H. WELCH, of the Johns Hopkins University, has given the Lane lectures at the Cooper Medical College, San Francisco, his subject being 'Infection and Immunity.'

WE learn from *Nature* that the Lancashire and Western Sea Fisheries Joint Committee has appointed Dr. J. T. Jenkins, professor of biology in Hartley University College, South-

ampton, to be superintendent of sea fisheries in place of the late Captain Dawson.

Dr. Frederick B. Loomis, of Amherst College, has during the present summer been engaged in exploring for fossils in Wyoming.

A life of Andrée is to be issued by the Geographical Society of Stockholm, of which he was a prominent member.

Mrs. Imogen W. Eddy, computer in the Harvard College Observatory, was killed by a fall from an elevator on September 4.

The death is announced of Dr. Frederic Zahn, formerly professor of anatomy at the University of Geneva; and of Dr. Emilio Valleri, professor of experimental physics at Naples.

Dr. Alexander Graham Bell has presented to the Station for Experimental Evolution of the Carnegie Institution, at Cold Spring Harbor, Long Island, a six-nippled ram and two five-nippled ewes, one white and one black, from the remarkable race of multi-nippled sheep that he has created on Cape Breton Island. The sheep will be used to test the method of inheritance of characters.

The late Dr. Albert W. Warden, of Union Hill, West Hoboken, has bequeathed $1,000 to the New York Academy of Medicine.

The Peruvian government is about to found a national museum of natural science, which will have its home in Lima, and in which there will be three departments, one devoted to animal life, another to plant life, and a third to minerals.

The British antarctic expedition steamer *Discovery* arrived at Plymouth on September 10.

It is announced that a powerful steamer will be built immediately at the McKay & Dix shipyard at Verona, on the Penobscot, for Commander Peary's next polar expedition. The steamer will be equipped with powerful engines and it is said that some new features will be embodied in the construction. It will cost between $300,000 and $400,000.

According to a press despatch the Ceylon mineralogical survey report gives details of the discovery of a mineral existing in small black cubical crystals in the refuse of the gem washings at Balangoda. The principal constituent is thorium oxide, of which there is present more than 75 per cent. The mineral appears to be new, and it is suggested that the name of thorianite be given to it, as it is radio-active.

There will be a civil service examination on September 14 for the position of aid and deck officer in the Coast and Geodetic Survey and one for the position of computer on September 19. Deck officers are appointed with salaries of $720 to $900 and computers receive salaries of from $1,000 to $2,000. Further information may be obtained from the active superintendent of the survey. Applications for the position of scientific aid (female) in the Bureau of Plant Industry, Department of Agriculture, will be received by the Civil Service Commission until September 28, 1904. Applicants must be graduates of colleges in courses of study tending to qualify them for the scientific work of the Department of Agriculture. They are not assembled for this examination, but must submit all the required material with their applications.

The U. S. Geological Survey has in the government building of the St. Louis Exposition an exhibit of radio-active substances, prepared under the direction of Dr. George F. Kunz, which will be of interest to scientific men who visit the exposition. It includes specimens from M. Becquerel, Sir William Crookes and Professor Ernest Rutherford and others. Everything obtainable relating to the source, manufacture and application of radium is exhibited, including all chemicals obtained from the separation of various radium compounds, and all instruments and devices with which it is proposed to apply radio-activity in medicine, science and the arts. A feature is the portraits and the publications of investigators, together with photographs of their laboratories and apparatus, and autograph letters from some of them. Two halls have been set aside for demonstration of the properties of radium. In one are grouped the specimens of ores and minerals containing radium. In the other illustrated lectures are given daily at 11:30 A.M. and 3 P.M. on a variety of subjects re-

lating to the history, nature and possibilities of radium. Its mode of occurrence, the methods used in separating it from its ores, the concentration of activities from low to high, and the manifold uses to which these remarkable radio-active substances may be put are described. Cinematograph Hall can be darkened, and different highly active specimens of radium compounds are exhibited as affecting the diamond, willemite, kunzite and other radio-responsive substances.

AT the last monthly general meeting of the Zoological Society of London the report of the council for July was read by the secretary, Dr. P. Chalmers Mitchell. It was stated that during the month 180 additions had been made to the society's menagerie. The total number of visitors to the society's gardens during the year up to the end of July was 417,170, an increase of 48,738 on the numbers for the corresponding period of last year. The number of fellows elected and readmitted during the year up to the end of July was 165, an increase of 31 as compared with the corresponding period of 1903.

Nature states that with the view to obtain further information on the growth and migrations of salmon (including sea-trout, salmon-trout, peal, sewin, etc.), the British Board of Agriculture and Fisheries had had a number of such fish 'marked' by attaching a small oblong silver label (oxidized, or blackened, and bearing distinctive letters and numbers) to the dorsal or large back fin. Small rewards will be paid for the recovery of fish bearing such labels or other 'marks,' or for information respecting them. The Board has prepared lists of persons in the south and west of England, in Wales and Monmouthshire, and in the north of England, who will receive marked fish. The experiments will be continued during a series of years, and the co-operation of net-fishermen, anglers, fishmongers, and all interested in the improvement of the salmon fisheries, is invited in order that the fullest possible results may be secured.

THE London *Times* says that the first instalment of specimens and a number of scientific instruments have arrived in Edinburgh from the *Scotia*. Many of the instruments were of so delicate a character that they had to be carried unpacked in the hands of those assisting in the transfer. Of specimens there were transshipped about 200 barrels, cases and packages. It is calculated that several years will be occupied in the classification of the immense amount of animal and oceanographic specimens which Dr. Bruce has brought home with him, and the contents of the great casks which contain the larger specimens will, even in rough sorting, take up some considerable amount of time. The larger specimens of oceanic life, upon being taken from the casks in which they have traveled so far, are placed in a specially designed zinc tub filled with methylated spirits and fitted with a rubber-rimmed lid, forming an excellent method of preservation. It is officially stated that a report which has been published regarding the sale of the *Scotia* is inaccurate.

THE U. S. Geological Survey has recently perfected plans for the systematic collection and preservation of well records and samples. The demand for information regarding wells has become so great that the survey has decided to issue an annual publication containing a brief account of the wells bored each year. This report will be published as near the beginning of the year as practicable, and will be sent to all those who apply for it. It will contain the names and addresses of persons doing well work and will summarize the work done by them in the preceding year. In order to obtain data for such a year book, the survey wishes to make arrangements with well drillers and well owners to send samples and records to the survey's headquarters in Washington, D. C. On receipt of names of persons willing to save samples of well borings the Survey will at once send them supplies of canvas bags in which they may transmit them through the mails without paying postage. As these samples may be sent as often as the drillers visit the postoffice, there is no danger that they will accumulate and become burdensome. For the keeping of a log a convenient pocket memorandum book will also be provided. It is hoped that the drillers and well

owners of the country will appreciate the importance of the Survey's effort and will cooperate so heartily as to assure its complete success. It should appeal to them for several reasons: (1) Their names and work will be kept before a class of readers interested in well drilling. (2) Records of their work will be carefully filed in the office of the survey, and will be readily available to them at any time, so that in case their notes are lost they can be duplicated. (3) Their cooperation will aid materially in the study of the geologic structure of the United States, and will thus assist in obtaining knowledge which can not fail to be of ultimate benefit to well drillers. It may sometimes be necessary to regard the records as confidential. In such cases the information will be carefully guarded and used only under the conditions stipulated by the informant. The director of the survey will be glad to have an expression of opinion regarding this work from well owners and drillers, and will be grateful to them for any suggestions.

UNIVERSITY AND EDUCATIONAL NEWS.

THE courts have confirmed the bequest of $40,000 made by the late Dr. William H. Crim to the University of Maryland School of Medicine.

THE Cooper Medical College, of San Francisco, has received a bequest from Mrs. Pauline C. Lane, wife of the late Dr. C. L. Lane, which enables it to build a library.

IT is stated in *Nature* that Mr. Frederick Soddy has concluded a series of university extension lectures in Western Australia. The last lecture was delivered on July 23, and on this occasion the premier of the colony, Mr. Walter James, in proposing a vote of thanks to Mr. Soddy, referred to the desirability of establishing a university in Western Australia. During the course of the last twelve months one distinct step has been taken in advancing the movement by the passage of the University Endowment Act. Endowment trustees have been appointed, and in these trustees some 700 or 800 acres of land have been vested, which promise to give the future university the richest endowment enjoyed by any university in Australia. They were very apt to think, Mr. James continued, that no university could be established unless they first expended a large sum of money in an elaborate building. He wished only they could convince the residents of Western Australia that so long as they had efficient workshops for their professors, the sooner they commenced to get their professors the sooner could they begin the work of the university, without money overburdening it in the first instance. Mr. Soddy's visit has done good in bringing home more thoroughly than before how necessary it is that the establishment of this university should be commenced without undue delay.

THE position of assistant in bacteriology in the Laboratory of Hygiene, University of Pennsylvania, and the Thomas A. Scott fellowship in hygiene in the same laboratory are vacant for the year 1904–1905. Further information may be obtained from the director of the laboratory.

MR. WILL GRANT CHAMBERS has been called from the chair of psychology and education in State Normal School at Moorhead, Minn., to the chair of psychology in the State Normal School of Colorado, at Greeley.

DR. H. T. MARSHALL has been appointed professor of pathology at Baltimore Medical College.

L. C. KARPINSKI, A.B. (Cornell, '01), Ph.D. (Strassburg, '03), and John W. Bradshaw, A.M. (Harvard), Ph.D. (Strassburg, '04), have been appointed instructors in mathematics in the University of Michigan.

PROFESSOR LORRAIN SMITH, of Belfast, and Dr. A. S. Grünbaum, of Liverpool, have been appointed to chairs of pathology, respectively, at the Universities of Manchester and Leeds.

DR. H. BATTERMANN, astronomer at the Berlin Observatory, has been appointed professor of astronomy at the University of Königsberg.

PROFESSOR O. E. MEYER, director of the physical laboratory of the University of Breslau, will retire from active service.